Mussa Redux and Conditional PPP

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Abstract:

The extreme persistence of real exchange rates found commonly in post-Bretton Woods data does not hold in the preceding fixed exchange rate period, when the half-life was roughly half as large in our sample. This finding supports sticky price models as an explanation for real exchange rate behavior, extending the classic argument of Mussa (1986) from a focus on short-run volatility to long-run dynamics. Two thirds of the rise in real exchange rate variance observed across exchange rate regimes is attributable to greater persistence of responses to shocks, including greater price stickiness, rather than to greater variance of shocks themselves.

Keywords: real exchange rate, persistence, exchange rate regimes, sticky price models

JEL classification: F0, F15, F31

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1. Introduction

There is a long-standing and ongoing debate over what type of model best explains the behavior of the real exchange rate, including its volatility, persistence, and comovement with the nominal exchange rate. On the one hand, sticky price models argue that shocks to the nominal exchange rate are passed on to the real exchange rate because of sticky nominal prices. On the other hand, advocates of real models of the real exchange rate – “real-real models” – argue that movements in the nominal exchange rate primarily reflect effects passed on from shocks to the real exchange rate. In his 1986 Carnegie-Rochester paper, Michael Mussa offered an influential critique of real-real exchange rate models by observing that the volatility of the real exchange rate is higher under a flexible nominal exchange rate regime than under a fixed exchange rate regime, such as the Bretton Woods system. This finding often is used to support a sticky price story, as it is thought that the more flexible nominal exchange rates prevailing in the post-Bretton Woods period permitted a rise in the volatility of nominal shocks.

Since the time that Mussa wrote his paper, the study of real exchange rates has shifted from examining exchange rate volatility, what Mussa referred to as “short-term fluctuations,” toward the analysis of the long-run behavior of the real exchange rate.\textsuperscript{2} In particular, most recent research has focused on asking if deviations in the real exchange rate from its purchasing power parity (PPP) equilibrium level tend to disappear in the long run, and how long it takes the real exchange rate to return to its long run level. Many of these papers have criticized sticky price models for their inability to explain the evident persistence of real exchange rate deviations from PPP.\textsuperscript{3} As a recent example of this debate, Steinsson (2008) argued that the hump-shaped

\textsuperscript{2} In his analysis Mussa (1986) worked with data in first differences (at the quarterly frequency) and consistently referred to his results as “short-term” fluctuations.  
\textsuperscript{3} For a summary, see Rogoff (1996). For a demonstration of how sticky price models have worked to overcome this criticism, see Carvalho and Necchio (2010).
dynamics and long half-life of the real exchange rate in the flexible exchange rate period cannot
be explained by a sticky price model in terms of nominal shocks.

This paper develops an updated version of the Mussa critique. We ask whether recent
findings regarding persistent dynamics in the literature studying the standard post-Bretton
Woods dataset apply also to the preceding Bretton Woods period of generally fixed exchange
rates. In particular, we analyze whether the dynamics or half-life of the real exchange rate are
affected by a change in the nominal exchange rate regime. If the real exchange rate is driven by
real shocks, then a change in the regime governing the nominal exchange rate should not affect
the dynamics of the real exchange rate. But if the change in nominal regime does affect real
exchange rate dynamics, then sticky price models might be needed to provide an explanation.4

Our methodology takes advantage of recent advances in panel econometrics and applies
them to estimating the dynamic properties of the real exchange rate. Specifically, the method of
Pesaran (2006) is adapted to estimate an autoregression of the real exchange rate over the
Bretton Woods and post-Bretton Woods periods for a panel of 20 industrialized countries. This
methodology controls for contemporaneous correlation across country pairs in the panel. In
addition, a two-equation vector error correction model (VECM) is estimated, which decomposes
the real exchange rate into its nominal exchange rate and relative price components.

The key finding of the paper is that the dynamic properties of the real exchange rate
differ between the Bretton Woods and post-Bretton Woods periods. The half-life we estimate for
the fixed exchange rate period, roughly two years, is about half as long as that for the flexible

4 Mussa (1986) reported a variety of statistics for the nominal exchange rate and price indices as well as for the real
exchange rate, including the variance, covariance, mean, and serial correlation. Interestingly, in his list of empirical
observations, he notes a rise in persistence under flexible exchange rates: “Short-term changes in nominal exchange
rates and in real exchange rates show substantial persistence during subperiods when the nominal exchange rate is
floating.” We take it as reassuring that his observations coincide with our claim of greater persistence during the
post-Bretton woods period, even if his main focus was on short-run fluctuations rather than on long-run convergence
and half-lives of the real exchange rate to PPP equilibrium.
exchange rate period of about four years. The finding is surprising, as theories going back to Friedman (1953) maintain that a flexible exchange rate should be useful as an alternative adjustment mechanism of relative prices when nominal prices are not free to adjust. This would suggest that the adoption of a flexible exchange rate should reduce the persistence of real exchange rate dynamics rather than raise it. The finding that the nominal exchange rate regime affects the half-life of the real exchange rate offers a new stylized fact, for which models of the real exchange rate should account. Further, the VECM estimation indicates that much of the rise in real exchange rate persistence can be attributed to changes in the short-run dynamics of prices, which can be interpreted as a rise in price stickiness.

While these findings support Mussa’s general argument, they suggest a reinterpretation in two respects. First, the variance of the real exchange rate can be decomposed into an “extrinsic” component representing exogenous shocks, and an “intrinsic” component representing persistence in the dynamic response to shocks, borrowing the terminology of Obstfeld and Stockman (1985). Our estimates indicate that about two thirds of the increase in the variance of the real exchange rate observed by Mussa can be attributed to the greater persistence in the intrinsic component, rather than a rise in the variance of exogenous exchange rate shocks that was Mussa’s focus. Second, the VECM estimates indicate that a significant portion of this rise in real exchange rate persistence is due to a rise in the degree of price inertia associated with flexible exchange rates, rather than a change in the behavior of nominal exchange rates. One interpretation is that this result reflects the endogenous nature of price stickiness in the face of changing market conditions.

Although the results of this paper are of interest to the large literature estimating half-

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5 Recent theoretical contributions show that Friedman’s conclusions are conditional upon assumptions such as exports invoiced and sticky in the currency of the exporter. See Berka, Devereux, and Engel (2012) for a recent discussion of Friedman’s claims.
lives of the real exchange rate, this literature has focused on the modern era of flexible exchange rates; our paper differs in that it draws lessons by comparing across these two exchange rate regimes. Several studies use long horizons encompassing multiple regime periods as a means to enhance statistical power, but they do not compare half lives across regimes. Only a few papers have estimated half-lives for different exchange rate regimes, but they are not motivated by the question of evaluating real versus monetary models, and most do not use panel techniques.

Taylor (2002) is most similar in terms of its motivation. He finds only a small rise in half-lives in the post-Bretton Woods period, we argue, due to a shorter sample length and the presence of several developing countries in his sample. These countries confound a division of subsamples based on Bretton Woods dates, as some were de-facto floating for many years during the Bretton Woods period, and some returned to pegging after the end of Bretton Woods. Parsley and Popper (2001) use a sample consisting mainly of developing countries, and they do not allow for the changes in short-run dynamics that play an important role in our findings. Sarno and Valente (2006) differ in aggregating the Bretton Woods period together with the Gold Standard as two examples of fixed exchange rate regimes.

Our methodology for decomposing real exchange rate changes into their underlying components is closely related to Cheung, Lai, and Bergman (2004), but they are interested only in the flexible exchange rate period, and they do not employ panel techniques.

The data and preliminary analysis involving stationarity tests are presented in Section 2.

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6 See Imbs et al (2005) for a prominent example and discussion of this general literature.
7 For example, Abuaf and Jorion (1990) find average half-lives of 3.3 years for bilateral real exchange rates between the U. S. and eight countries for the period 1900 to 1972. Murray and Papell (2002) find an average half-life of 3.6 for six countries over 1900 to 1996. Frankel (1986) uses a 116-year-long dataset for the dollar-pound real exchange rate and reports a half-life of 4.6 years. Lothian and Taylor (1996) find a half live of 4.7 years for the dollar-pound rate using two centuries of data. In contrast to our analysis, none of these papers compares the behavior of the real exchange rate across periods.
8 See the de-facto regime classifications in Shambaugh (2004).
9 Kim (2012) applies VECM methodology to a long data sample that includes multiple exchange rate regimes, but it does not estimate over subsamples to compare results across regimes.
The main empirical results are in Sections 3 and 4, where Section 3 estimates half-lives from single equation autoregressions, and Section 4 explains this result by estimating a VECM and conditioning on shocks. Section 5 summarizes conclusions.

2. Data and preliminary analysis

The dataset consists of bilateral nominal exchange rates with the U.S. dollar as the numeraire and consumer prices indices, for 20 industrialized countries, taken from International Financial Statistics. The sample is annual in frequency and covers the period 1949 to 2010.

Define the real exchange rate, \( q_{jt} \), as the relative price level between country \( j \) and the base country (here the U. S. dollar) in period \( t \), computed as \( q_{jt} = e_{jt} + p_{jt} \), where \( e_{jt} \) is the nominal exchange rate (currency \( j \) per U.S. dollar), and \( p_{jt} = p_{US,t} - p_{j,t}^* \) is the log difference between the price indices in the United States and country \( j \), both in local currency units, and all variables are expressed in logs.

As preparation for the main analysis later, we first establish that international relative prices are stationary. We apply the cross-sectionally augmented Dickey-Fuller (CADF) test provided by Pesaran (2007). Consider the following regression:

\[
\Delta q_{jt} = a_j + b_j (q_{jt-1}) + c_j (\bar{q}_{t-1}) + d_j (\Delta \bar{q}_t) + \epsilon_{jt} \\
\text{for } j = 1, ..., N, \text{ and } t = 1, ..., T
\]  

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10 The full list of 20 countries is: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. Other studies of PPP with aggregate data during the post-Bretton Woods period use similar country samples (e.g. Papell 2002). The nominal exchange rate data are defined as yearly averages.

11 This specification assumes that \( p_{US,t} \) and \( p_{j,t}^* \) share similar convergence speeds, a property that has been found to be consistent with the data; see Cheung et al. (2004).
where $\bar{q}_i = \sum_{j=1}^{N} q_{j,i}$ is the cross-section mean of $q_{j,i}$ across the $N$ country pairs and $\Delta \bar{q}_i = \bar{q}_i - \bar{q}_{i-1}$.

The purpose for augmenting the cross-section mean in the above equation is to control for contemporaneous correlation among $\epsilon_{j,t}$. The null hypothesis can be expressed as $H_0 : b_j = 0$ for all $j$ against the alternative hypothesis $H_1 : b_j < 0$ for some $j$. The test statistic provided by Pesaran (2007) is given by

$$CIPS(N,T) = (1 / N) \sum_{j=1}^{N} t_j(N,T)$$

where $t_j(N,T)$ is the $t$ statistic of $b_j$ in equation (1).

The top panel of Table 1 indicates rejection of nonstationarity at the 10% level for all subsamples: the Bretton Woods sample, 1949-1971; the post-Bretton Woods sample, 1973-2010; and the whole sample combined. Further, the longer data length afforded by the whole sample rejects nonstationarity even at the 1% significance level.

3. Estimating rates of convergence

This section documents the distinct speeds of real exchange rate convergence toward stationarity for different exchange rate regimes, and considers some implications.

3.1. Estimation of a single equation autoregression

We begin by estimating an autoregressive model of real exchange rates with panel data. To control for contemporaneous correlation of residuals, the common correlated effects pooled

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12 The estimation start dates of the two subsamples are 1950 and 1974, respectively, to accommodate the lagged dependent variable; we omit the transition year 1972.
(CCEP) regressor of Pesaran (2006) is used to estimate the autoregressive coefficients of real exchange rates. More specifically, the estimation equation is:

\[ q_{j,t} = c_j + \sum_{m=1}^{M} \rho_m (q_{j,m}) + \epsilon_{j,t} \]  

augmented with cross-section means of the left-hand and right-hand variables (\( \bar{q}_t, \bar{q}_{t-1}, \ldots \bar{q}_{t-M} \)).

To control for potential bias in the CCEP estimator from the presence of lagged dependent variables, the standard double bootstrap procedure of Kilian (1998) is employed with 1000 replications to obtain bias-adjusted estimates. Specification tests based on a bias-adjusted version of the AIC indicate a median optimal lag length of one (\( M = 1 \)) for both subsamples. Results are also reported for two lags (\( M = 2 \)) to demonstrate that the conclusions are robust.

Table 2 reports coefficient estimates and half-lives of the real exchange rate, computed on the basis of simulated impulse responses. The half-life estimated for the fixed exchange rate Bretton Woods period is 2.27 years (with a 5%-95% band of 1.43 to 3.39 years). That for the more flexible exchange rate post-Bretton Woods period is 4.31 years (with a band of 3.44 to 5.24). The latter number is within the “consensus” range in the empirical literature of between 3

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13 As discussed in Pesaran (2006), the cross-sectional means are observable proxies for the common effects in the panel. STATA code to conduct CCEP estimations used throughout the paper are available upon request.

14 See the appendix of Bergin, Glick, and Wu (2013) for a Monte-Carlo study of the bias of the CCEP estimator when applied to models with a lagged dependent variable. In our bootstrap methodology we do block resampling of residuals (recentered by currency pair) with replacement and initialize with actual data. We use block sizes of two periods; the results are not affected by using different block sizes.

15 The bias-adjusted version of AIC, AICC, first proposed by Hurvich and Tsai (1989), takes account of small sample biases in autoregressive time series models. We applied the AICC to the residuals of individual AR equations estimated for each currency pair individually. Of the 20 countries in the sample, 19 had an optimal lag length of 1 for the Bretton Woods period (with just Italy having an optimal lag length of 2), and 15 of 20 have an optimal lag length of 1 for the post-Bretton Woods period (with 2 lags optimal for the U.K., Belgium, Denmark, Portugal, and New Zealand).

16 The half-life is computed as the time it takes for the impulse responses to a unit shock to equal 0.5, as defined in Steinsson (2008). We identify the first period, \( t_1 \), where the impulse response \( f(t) \) falls from a value above 0.5 to a value below 0.5 in the subsequent period, \( t_1 + 1 \). We interpolate the fraction of a period after \( t_1 \), where the impulse response function reaches a value of 0.5 by adding \( (f(t_1) - 0.5)/ (f(t_1) - f(t_1+1)) \).
to 5 years. We find that the half-life under fixed exchange rates was distinctly smaller than under the flexible rates, being just about half the size. A similar result obtains for the specification of two lags in the autoregression: with half-lives of 2.75 and 4.27, respectively.

Figure 1 plots the impulse responses associated with these half-lives. Observe in the lower panel that when two lags are included in the autoregression a hump-shaped response can be detected for the post-Bretton Woods data, as observed in Cheung, Lai, and Bergman (2004) and Steinsson (2008). However, the upper panel reveals no hump-shaped response in the Bretton Woods period. If the hump-shaped dynamics are a reason for the high degree of persistence in recent real exchange rate data, as Steinsson conjectures, then the lack of this hump may help explain the lower degree of persistence in the Bretton Woods period.

3.2. Tests of significance

To confirm statistical significance, the nested specification of Parsley and Popper (2001) is adopted next, which combines the data samples from multiple sub-periods. The estimation equation specifies a single autoregressive coefficient over both samples, where an indicator regime variable \( d_t \) takes a value of 0 for years during the Bretton Woods period (1949-1971), and a value of 1 for years afterward (1973-2010).

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17 See Rogoff (1996) and Imbs et al (2005). Our estimated half-life for the post-Bretton Woods period is larger than that found in Bergin, Glick, and Wu (2013). As explained in that paper, the faster convergence speed is attributable to the use of product-level data that restricts the sample to the period 1990-2007.

18 As reported in the online appendix, results are very similar if we limit the sample to the 14 countries with optimal lag length of 1 for both periods. Results are inconclusive if we limit the sample to the set of 5 countries with optimal lag length of 1 in the Bretton Woods period and 2 in the post-Bretton Woods period.

19 The estimated equation also includes the cross-sectional means of the dependent and explanatory variables as well as the regime indicator \( d_t \), which we treat as an observed common effect entering the \( \bar{H} \) matrix in the CCEP estimator in Pesaran (2006). In implementing the Kilian (1998) procedure to control for potential estimator bias, we do block resampling of residuals (recentered by currency pair for each regime period) with replacement and initialize with actual data.
In this specification the coefficient \( \rho_{n,d} \) measures the change in persistence in the real exchange rate after the end of the Bretton Woods period. Table 3 reports coefficient estimates and half-life estimates for various specifications of lags \((M=N=1, M=N=2, M=1 \text{ and } N=2)\). In all cases the estimate of \( \rho_{n,d} \) is positive and statistically significant. Further, the half-lives estimated are uniformly higher in the post-Bretton Woods period, with confidence bands not overlapping with those of the Bretton Woods period.\(^{20}\) Compared to the specification in equation (2), equation (3) has the advantage of nesting both periods within one specification, thus allowing for direct tests of statistical significance for the change between periods. However, a disadvantage of this nested approach is that it assumes the same error distribution for both regimes; this is not true for the specification in equation (2) which is estimated for each period separately.

The shorter half-life during the Bretton Woods period is surprising, as theories dating back to Friedman (1953) posit that a flexible exchange rate should be useful as an alternative adjustment mechanism of relative prices when nominal prices are not free to adjust. This suggests that the imposition of a fixed exchange rate should raise the half-life of the real exchange rate rather than lower it. It is also surprising in the context of nonlinear models like Taylor, Peel, and Sarno (2001) and Sarno and Valente (2006), since they predict that large shocks should be corrected more quickly, and nominal exchange rate shocks tend to be larger in magnitude than aggregate price shocks.

\(^{20}\) These conclusions also apply equally well to a case not reported in the table, the case with two lags for the Bretton Woods sample and one lag for post-Bretton Woods. This case can be accommodated within the specification of equation (3) by specifying \(M=N=2\) along with the restriction that \(\rho_{2,d} = -\rho_2\). The estimate of \(\rho_{1,d}\) (0.067) is positive and statistically significant at the 5% level, and the half-life estimate for post Bretton Woods (6.87) is greater than that for Bretton Woods (2.40) with 5%-95% confidence bands not overlapping.
3.3. **Implications for real exchange rate variance**

The finding of greater persistence after Bretton Woods ended also provides a new interpretation for the long-standing finding regarding the rise in the variance of the real exchange rate in this period. The variance of the real exchange rate can be decomposed into the extrinsic and the intrinsic components, to borrow the terminology from Obstfeld and Stockman (1985). The former represents the exogenous shock to the autoregression in equation (2), and the latter is the endogenous propagation characterized by the dynamic parameters $\rho_m$ in that equation. If one transforms equation (2), in the case of a single lag, by subtracting the mean of the real exchange rate from each side, squaring, and taking expectations, one derives:

$$
\text{var}(q_{jt}) = \left(\frac{1}{1 - \rho_t^2}\right) \cdot \text{var}(\varepsilon_{jt}).
$$

(4)

This formula provides a clean decomposition of the variance of the real exchange rate, where $\left(1/(1 - \rho_t^2)\right)$ is the contribution of the intrinsic component, and $\text{var}(\varepsilon_{jt})$ is the extrinsic component. Using the estimates of $\rho_t$ from columns (1) and (2) in Table 2, we compute that the term $\left(1/(1 - \rho_t^2)\right)$ increased by 66%. For a comparison, consider that the variance of the real exchange rate, $\text{var}(q_{jt})$, among countries in our sample doubled in the post-Bretton Woods sample compared to the Bretton woods sample: the $\text{var}(q_{jt})$ averaged across countries was 0.0123 under Bretton Woods and 0.250 afterward, a rise of 102.45%.\(^{21}\) This comparison suggests that two thirds of the rise in real exchange rate variance under flexible exchange rates in

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\(^{21}\) Using medians across countries rather than means indicates a larger rise in average $\text{var}(q)$ of 205%, as there is substantial heterogeneity in the level of $\text{var}(q)$ in the Bretton Woods sample. Both computations suggest that the intrinsic component provides a substantial contribution to the overall rise in variance.
our data was due, not to a rise in the extrinsic shocks to the real exchange rate, but instead to the rise in persistence under the flexible rate regime.22

This result is robust when applied also to the other estimations and samples discussed previously in the paper, although the derivation of the decomposition is more complicated than for the simple autoregression with one lag. Applying the decomposition to the autoregression estimated with two lags, i.e. \( M = 2 \), equation (2) implies the following (see the online appendix for explanation):

\[
\var(q_{jt}) = \Psi \cdot \var(\varepsilon_{jt}), \quad \text{where} \quad \Psi = \frac{1}{1 - \rho_1^2 - \rho_2^2 - \left(2 \rho_1^2 \rho_2 / (1 - \rho_2)\right)}.
\] (5)

Substituting in the autoregressive parameter values from the AR(2) estimation, the result is that \( \Psi = 2.68 \) for the Bretton Woods period and 3.95 in the post-Bretton Woods period, a rise of 47%. When applied to the estimates from the nested regression specification (3), the implied rise in intrinsic component is 146% for the specification with one lag, a rise of 80% for the specification with two lags, and a rise of 151% for the third case with mixed lags.

3.4. Relationship to earlier estimates

Our results differ from Taylor (2002), who found only a modest increase in the half-lives of countries between the Bretton Woods period and the floating period that followed. He reports estimates pooled across countries as well separately for each country; since our panel methodology implicitly pools across countries, the pooled estimates in Taylor are the most appropriate comparison. Taylor’s pooled estimates of the half-life rose modestly from 1.5 to 2.1

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22 Our analysis is based on using real exchange rate data in level form rather than first differenced or filtered data. Given that we have already rejected the null hypothesis of nonstationarity of the real exchange rate for this sample, it is appropriate to use the levels data.
years. (When he estimated separately by country, the median half-life among his sample
countries rose from 2.1 years to 2.6 years, while the mean rose only from 2.4 to 2.6 years.)

Taylor graciously provided his dataset to us, enabling us to investigate the differences in results.
The second row of Table 4 reports that when we apply our estimation methodology to Taylor’s
dataset, we find the same values as did Taylor: we estimate a half-life of 1.5 for Bretton Woods,
and a half-life of 2.1 for the floating period.\(^{23}\) We conclude that our particular methodology,
Pesaran’s CCEP panel estimation with bias correction, is not the source of difference in our main
results.\(^{24}\)

Next we trace where the difference does come from. Our dataset studies only developed
economies, whereas Taylor includes three developing economies: Argentina, Brazil and
Mexico.\(^{25}\) Many developing economies, including those in Taylor’s data, do not fit the definition
of our subsamples, which associate Bretton-Woods years with a dollar peg regime and post-
Bretton Woods years with the absence of a dollar peg. Rather, these countries often switch
between fixed and flexible regimes over the sample period, regardless of Bretton Woods dates.\(^{26}\)
Row 3 in Table 4 shows that when we exclude these three developing countries from Taylor’s
sample and continue to apply our estimation methodology, we find a higher half-life for the

\(^{23}\) For comparison, we follow Taylor (2002) in reporting values to one decimal place.
\(^{24}\) This should also be reassuring to Taylor, as our methodology has the advantage of controlling for
contemporaneous correlations of error terms in the cross-section, which can be a source of significant bias in
country pair panel datasets.
\(^{25}\) Taylor’s sample also differs from ours in excluding Austria, Greece, Ireland, and New Zealand.
\(^{26}\) As an example, the de-facto regime classification of Shambaugh (2004) indicates Argentina did not peg to the
dollar for more than half of the Bretton Woods years in his sample (7 out of 12 years); further, Argentina did peg to
the dollar for more than a third of the post-Bretton Woods years in his sample (12 out of 32 years). A similar, if less
extreme, characterization applies to the other two developing countries in Taylor’s sample: Mexico continued to peg
to the dollar in many years after the end of Bretton Woods, and Brazil floated during some of the Bretton Woods
years.
floating period, 3.1 years, compared to that for Bretton Woods, 1.9 years. This gap is widened yet further when we extend Taylor’s sample period to 2011 as in our dataset, with half-lives of 1.9 for Bretton Woods and 3.6 for the floating period. These changes in the dataset are sufficient to explain a sizeable gap between the half-lives of the two periods.

As shown in the table, the remaining difference between these estimates and our benchmark results is due (1) to the fact that our dataset uses period averages to measure the exchange rate, as is common practice in the literature, and (2) the fact our data from IFS starts in 1949 rather than 1946 as in Taylor’s dataset. The first of these points supports the conjecture in Taylor (2001) that the common practice of using end period values for the exchange rate, due to data limitations in the IFS dataset, can affect estimates of half-lives. Nonetheless, our estimates in Table 4 indicate that finding a large gap in half-lives between the two time periods does not depend upon this convention.

Our main result differs from Parsley and Popper (2001) who find faster convergence in the post-Bretton Woods period. They estimate a model of real exchange rate adjustment with a panel of 82 developing and developed countries for the period 1961 to 1992. This result is attributable to the predominance of developing countries in their sample, many of which maintained pegged rates in the post-Bretton Woods period through the end of their sample in 1992.

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27 We note that in Taylor’s non-pooled estimates for each country separately, when one takes an average excluding the three developing economies, this raises the half-lives of both periods, but without creating a difference between periods. We conclude that the three developing economies must have a large effect on the pooled estimate in comparison to the simple average of the non-pooled estimates with each country receiving equal weight.

28 Taylor’s dataset for these extra years appear to be based upon interpolation of missing values.

29 Their sample includes 17 of the 20 industrial countries in our sample, excluding Greece, Ireland, and Portugal.

30 In fact, when classifying countries by their individual exchange rate regimes, they find that countries pegged against the dollar adjusted more quickly than those with more flexible regimes, consistent with our finding. There are several key methodological differences between Parsley and Popper (2001) and our paper that also should be
Our main result also differs from Sarno and Valente (2006), who find faster convergence under fixed exchange rate regimes for the four dollar currency pairs in their sample (for France, Germany, Japan, and the United Kingdom). However, this difference appears to arise from the fact they report results combining the Bretton Woods period together with the Gold Standard, whereas we focus on Bretton Woods alone for our fixed rate regime period. While Sarno and Valente (2006) do not report half-lives separately for Bretton Woods and the Gold Standard, Taylor does report separate results for each regime. His estimates pooling across countries indicate a half-life twice as long under the Gold Standard as under Bretton Woods, 3.1 years versus 1.5 years. This suggests that the relatively higher half-life in Sarno and Valente could arise from the fact they chose to aggregate over these two fixed-exchange rate periods.

4. Decomposing dynamics

We now investigate the source of the increase in persistence, using a vector error correction model (VECM). This approach permits one to decompose the dynamics of the real exchange rate into that of its two underlying components, the nominal exchange rate and the price level differential.\textsuperscript{31}

\textsuperscript{31} We employ this methodology in Bergin, Glick, and Wu (2013), which documents the asymptotic properties of this estimator for a vector error correction model and describes a bootstrapped bias correction approach suggested by Kilian (1998). Our results employ this bias-corrected estimation methodology. As with the autoregression estimation, this involves bootstrapping by block resampling of residuals (recentered by currency pair) with replacement and initializing with actual data. We use block sizes of three periods in view of the AR(2) nature of the specification in levels.
4.1. Estimation of a vector error correction model

The stationarity of real exchange rates implies cointegration of nominal exchange rates \((e_{jt})\) and relative prices \((p_{jt})\) with the cointegrating vector \((1, 1)\). The adjustment process of nominal exchange rates and relative prices can be studied using the following panel vector error-correction model (VECM):

\[
\Delta e_{jt} = \alpha_{e,j} + \rho_e (q_{j,t-1}) + \mu_e^e (\Delta e_{j,t-1}) + \mu_e^p (\Delta p_{j,t-1}) + \zeta_e^{ce}
\]

(6a)

\[
\Delta p_{jt} = \alpha_{p,j} + \rho_p (q_{j,t-1}) + \mu_p^e (\Delta e_{j,t-1}) + \mu_p^p (\Delta p_{j,t-1}) + \zeta_p^{cp}.
\]

(6b)

This two-equation system decomposes the real exchange rate, \(q_{j,t}\), into the nominal exchange rate, \(e_{jt}\), and the relative price level, \(p_{jt}\). It regresses the first difference of each of these components on the lag level of the real exchange rate, which summarizes the degree to which the data deviate from PPP. Other regressors in (5) control for level effects and short-run dynamics of the variables. The coefficients \(\rho_e\) and \(\rho_p\) reflect how strongly the exchange rate and prices respond to PPP deviations. Because negative movements in these variables work to reduce PPP deviations, they provide a measure of the speed of adjustment of nominal exchange rates and relative prices, respectively. To allow for possible cross section dependence in the errors, we computed CCEP estimators of the parameters by including as regressors the cross section averages of all variables \((\Delta \bar{e}_t, \bar{q}_{t-1}, \Delta \bar{e}_{t-1}, \text{ and } \Delta \bar{p}_{t-1})\) and \((\Delta \bar{p}_t, \bar{q}_{t-1}, \Delta \bar{e}_{t-1}, \text{ and } \Delta \bar{p}_{t-1})\) for the \(\Delta e_{jt}\) and \(\Delta p_{jt}\) equations, respectively.

Table 5 reports coefficients and half-lives of the real exchange rate conditional on shock. Almost all coefficients are statistically significant, and there are some intriguing differences in

---

32AICC indicated a median optimal lag length of 1 for the variables in first differences, as specified here.
point estimates between the two periods. The speed of adjustment parameters in both the $e$ and $p$
equations are smaller in absolute value in the post-Bretton Woods period. There also appear to be large changes in percentage terms in the short run dynamics parameters in the price equation. The statistical significance and quantitative effects of these changes between the two periods will be investigated further below.

The VECM also provides a basis for identifying distinct shocks to the system. We use a Cholesky ordering of the variables $e$, then $p$, which identifies as an exchange rate shock any innovation in the nominal exchange rate that is not explained as an endogenous response to the lagged values in the regression equation (6a). A price shock is then identified as an innovation in the price level not associated with a contemporaneous movement in the exchange rate. It has an advantage in the present context in that it avoids imposing an assumption of price stickiness (implying no contemporaneous movement in price), but rather allows the data to speak about the degree of price rigidity in response to shocks.33

Figures 2 and 3 plot impulse responses to the two shocks in each sample period, and Table 5 reports the half-lives of the real exchange rate for each shock based on these impulse responses. While the half-lives conditional on both shocks rise in the post-Bretton Woods period, those conditional on price shocks rise by a much larger percentage. Figure 2 shows the development of a small hump in the exchange rate and a lower adjustment to an exchange rate shock in the Bretton Woods period. Figure 3 shows the dramatic exaggeration of the hump in price dynamics and slower adjustment in response to a price shock.

4.2. Tests of significance

33 See Bergin, Glick, and Wu (2012) for results under the alternative Cholesky ordering, which in most respects are similar.
To facilitate tests of significance for changes in parameters between periods, we again extend the nested estimation specification of Parsley and Popper (2001). Our specification differs from theirs in allowing interactions of the period dummy with short run dynamics as well as with the speed of adjustment coefficients. In the expanded VECM specification below, $d_t$ again is an indicator variable that takes a value of 0 for years during the Bretton Woods period, and a value of 1 for years afterward:

$$
\begin{align}
\Delta e_{jt} &= \alpha_{e,j} + \rho_{e} q_{j,t-1} + \rho_{e,d} \left( d_t q_{j,t-1} \right) + \mu_{e}^c \Delta e_{j,t-1} + \mu_{e,d} \left( d_t \Delta e_{j,t-1} \right) \\
&+ \mu_{e}^p \Delta p_{j,t-1} + \mu_{e,d}^p \left( d_t \Delta p_{j,t-1} \right) + \zeta_{e,j,t} \\
\Delta p_{jt} &= \alpha_{p,j} + \rho_{p} q_{j,t-1} + \rho_{p,d} \left( d_t q_{j,t-1} \right) + \mu_{p}^e \Delta e_{j,t-1} + \mu_{p,d}^e \left( d_t \Delta e_{j,t-1} \right) \\
&+ \mu_{p}^p \Delta p_{j,t-1} + \mu_{p,d}^p \left( d_t \Delta p_{j,t-1} \right) + \zeta_{p,j,t} 
\end{align}
$$

(7a) (7b)

The results in Table 6 indicate there are changes in several coefficients between periods that are statistically significant. The first is the drop in the speed of adjustment in the nominal exchange rate equation ($\rho_{e,d}$), which clearly has the potential to increase the persistence of real exchange rate deviations.

The other two changes involve the short run dynamics of prices. The negative value of $\mu_{p,d}^e$ indicates that prices have a smaller (in absolute value) response to nominal exchange rate changes in the post-Bretton Woods period. This represents a form of price stickiness, governing the degree to which a currency depreciation is followed by an offsetting rise in domestic currency prices, for example. In the usual Mussa story, this is a key mechanism governing the speed at which the effect of nominal shocks on the real exchange rate die out.

---

34 Confidence bands used to determine significance are computed using the double-bootstrap method of Kilian (1998). Due to the nonstandard distribution of the coefficients, conventional t-tests are not useful, but standard errors are reported for reference.
The other change in short run dynamics is an increase in the autoregressive parameter on the first difference of prices (a positive $\mu^{p}_{p,d}$), which indicates a rise in inflation persistence.

The observation that inflation persistence rose in post-Bretton Woods data is a common stylized fact documented previously in the literature (e.g. see Alogoskoufis, 1992; Obstfeld, 1995). Further, it has been linked by some to the change in nominal exchange rate regime, arising from the greater monetary policy autonomy after the end of Bretton Woods. However, this stylized fact has not been considered previously as a contributor to greater real exchange rate persistence, and the intuition for this parameter’s effects on the real exchange rate is not as obvious as it was for the two parameters discussed above. Stochastic simulations will be used below to identify the mechanism by which inflation persistence alters real exchange rate dynamics.

4.3. Stochastic simulations of the VECM system

The next objective is to gauge the quantitative contributions of these changes in parameters to the rise in real exchange rate variance. Unlike the simple autoregression in equations (4) and (5), there is no log-linear expression permitting an analytical decomposition, so instead we conduct stochastic simulations to measure the decomposition. The VECM system (6a,b) is simulated with different combinations of dynamic coefficients drawn from the Bretton Woods and post-Bretton Woods estimations in Table 5, with shocks drawn from the Bretton Woods period.35 The simulation produces a time series of artificial data, from which one can

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35 The VECM system (6a,b) is used rather than (7a,b) because it does not impose the same error distribution in the Bretton Woods and post-Bretton Woods periods. Otherwise, this would rule out by assumption a role played by the extrinsic component in the decomposition of real exchange rate variance. The simulation begins with a random draw of 43 pairs of residuals collected from the estimation of the $e$ and $p$ equations in (6a,b) for the Bretton Woods period. These draws of residuals are fed sequentially into the system (6a,b), using the coefficient values in Table 5. The first 20 years of simulation are used to establish starting values for the endogenous variables, with the last 23 years representing the Bretton Woods period. Each simulation produces artificial time series for $p$, $e$, and hence $q$, and we
compute the variance of the real exchange rate. As the benchmark for comparison, the simulation is conducted first for the case where all 6 dynamics parameters \((\rho_c, \mu_c, \mu^p, \rho_p, \mu^p, \mu^p)\) are set at their values estimated from the Bretton Woods period. The simulation is then repeated six times, where each of the dynamics parameters in turn is set to its value in the post-Bretton Woods period, with all other parameters remaining at their Bretton Woods values. Further, for help with intuition later, impulse responses and half-lives are computed for each configuration of parameter settings.

The experiment indicates that changing all of the dynamics coefficients to their post-Bretton Woods estimated values raises the variance of the real exchange rate by 50.1% compared to the benchmark case using Bretton Woods parameter values. In the terminology used to describe equations (4) and (5) above, this rise in persistence is due to the intrinsic component, and it is similar in magnitude to the values of 66% and 47% estimated for the first order and second order autoregressions in Section III.

Table 7 reports the effects of changing each parameter separately. Consider first changes in the three parameters found to be statistically significant in Table 6. If only the speed of adjustment parameter in the error correction response in the exchange rate equation \((\rho_c)\) is set at its post-Bretton Woods value, with all other parameters coming from the Bretton Woods period, this parameter change can explain a 9.5% rise in real exchange rate variance. While part of the story, a fall in the adjustment speed of exchange rates is clearly not the whole story.

When changing the response of inflation to nominal exchange rate changes \((\mu^p)\), we find this change can explain an 11.8% rise in real exchange rate variance. This parameter change
works by raising real exchange rate persistence specific to a nominal exchange rate shock. Table 7 reports that the half-life conditional on $e$ shocks rises 0.35 years (from a half-life of 2.03 years under Bretton Woods to a half-life of 2.38 in the post-Bretton Woods period), while the half-life conditional on $p$ shocks in Table 7 instead falls. In particular, it works by cutting in half the initial rise in domestic prices in the period immediately after a nominal currency depreciation. Although the impulse responses for the counterfactual simulations are not reported, the reader can observe this effect in the VECM results of Figure 2, in that prices fall much less in year 2 of the impulse response during the post-BrettonWoods period in the bottom panel than they do in year 2 of the Bretton Woods period in the top panel.

Next, the increase in inflation persistence arising from the larger value of $\mu_p^*$ in the post-Bretton Woods period raises the variance of the real exchange rate by 12.8%. We know this parameter change works through price shocks, as the half-life conditional on price shocks rises an impressive 1.35 years (from a half-life of 1.56 years for the Bretton Woods period to 2.91 in the later period), whereas the half-life conditional on exchange rate shocks falls slightly. In particular, it makes price levels continue to rise further in periods following a positive price shock, leading to deviations from PPP that grow over time before diminishing as the error correction response dominates. This mechanism can be observed in the prominent hump-shaped impulse response of $p$ to a price shock in the lower panel of Figure 3.

Overall, the changes in price dynamics, including both a rise in price stickiness in response to nominal exchange rate shocks and a rise in inflation persistence in response to price shocks, account for a rise in real exchange rate variance of 24.6%, which is much larger than the
9.5% implied by the change in speed of adjustment of the nominal exchange rate.\textsuperscript{36} We conclude that more of the rise in real exchange rate persistence and volatility is attributable to a rise in persistence in price dynamics than due to a change in nominal exchange rate dynamics. One interpretation of this finding is that it reflects the endogenous nature of price stickiness, which could have risen in response to the change in environment created by the switch in exchange rate regime. The introduction of a flexible nominal exchange rate as an alternative source of relative price adjustment might have made less imperative the adjustment of goods prices in response to shocks.

5. Conclusions

Exchange rate regimes affect the long-run dynamics and half-life of the real exchange rate. The key finding is that the stylized fact of a very slow rate of convergence of the real exchange rate to PPP during the post-Bretton Woods period does not hold for the Bretton Woods period for our sample. Further, VECM analysis indicates the majority of this rise in real exchange rate persistence lies in a change in the short run dynamics of prices, including a rise in price stickiness and inflation persistence.

These findings have implications for the ongoing and fundamental debate over the source of real exchange rate fluctuations. Mussa used the fact that short-run real exchange rate volatility was affected by the nominal exchange rate regime to argue in favor of nominal shocks as an important driver of real exchange rates. In the same way we counter recent arguments

\textsuperscript{36} Some of the remaining rise in variance in the simulation is due to a lower estimate of the error correction response of prices to real exchange rate deviations ($\rho_e$). While the statistical analysis in Table 6 failed to show this parameter change was statistically significant, simulations using the point estimates indicate it would account for a 9.5% rise in $q$ variance. Since the changes in parameters governing exchange rate short-run dynamics ($\mu_e$ and $\mu_p$) each lower the variance of the real exchange rate in simulations rather than raise it, the remainder of the rise in variance unexplained by the individual parameter changes is due instead to positive interactions among the parameters.
against the role of nominal shocks in explaining real exchange rate persistence by pointing to the observation that real exchange rate dynamics are also affected by the nominal exchange rate regime.
6. Acknowledgements

We thank Alec Kennedy and Jeremy Pearce for research assistance. The views expressed below do not represent those of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.
7. References


<table>
<thead>
<tr>
<th>Sample</th>
<th>b</th>
<th>t-stat</th>
<th># pairs</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bretton Woods</td>
<td>-0.3322</td>
<td>-2.138</td>
<td>20</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Post-Bretton Woods</td>
<td>-0.2397</td>
<td>-2.132</td>
<td>20</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Whole sample</td>
<td>-0.1742</td>
<td>-2.496</td>
<td>20</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Note: The table reports estimates of the equation

$$\Delta q_{jt} = a_j + b_j (q_{jt-1}) + c_j (\bar{q}_{jt-1}) + d_j (\Delta \bar{q}_t) + e_{jt}; \quad j = 1,...,N \text{ and } t = 1,...,T$$

where $\bar{q}_t = \sum_{j=1}^{N} q_{jt}$ is the cross-section mean of $q_{jt}$ across country pairs and $\Delta \bar{q}_t = \bar{q}_t - \bar{q}_{t-1}$. The last three columns report whether the null hypothesis of nonstationarity can be rejected at a given level of significance.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>one lag</td>
<td>two lags</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bretton</td>
<td>Post-Bretton</td>
<td>Bretton</td>
<td>Post-Bretton</td>
</tr>
<tr>
<td></td>
<td>Woods</td>
<td>Woods</td>
<td>Woods</td>
<td>Woods</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.737***</td>
<td>0.852***</td>
<td>0.879***</td>
<td>1.022***</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.018)</td>
<td>(0.107)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>--</td>
<td>--</td>
<td>-0.115</td>
<td>-0.190***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.097)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>Half-life of $q$:</td>
<td>2.27</td>
<td>4.31</td>
<td>2.75</td>
<td>4.27</td>
</tr>
<tr>
<td>(5%, 95%)</td>
<td>(1.43, 3.39)</td>
<td>(3.44, 5.24)</td>
<td>(1.65, 4.71)</td>
<td>(3.51, 5.09)</td>
</tr>
<tr>
<td>(10%, 90%)</td>
<td>(1.57, 3.02)</td>
<td>(3.63, 4.97)</td>
<td>(1.78, 3.98)</td>
<td>(3.68, 4.88)</td>
</tr>
</tbody>
</table>

Note: Estimates are based on the equation $q_{jt} = c_j + \sum_{m=1}^{M} \rho_m (q_{jt-m}) + \varepsilon_{jt}$ along with cross-sectional means of all dependent and explanatory variables. Half-lives in years are calculated from simulated impulse responses derived from the parameter estimates. Bias correction is carried out via the Kilian (1998) bootstrap method with 1000 iterations. Standard errors in parentheses and p value are derived using the double-bootstrap method of Kilian (1998). *** indicates statistical significance at the 10% level, ** at 5%, * at 10%.
### Table 3: Nested Autoregression Estimates and Real Exchange Rate Half-lives

<table>
<thead>
<tr>
<th></th>
<th>(1) One lag $(M=N=1)$</th>
<th>(2) Two lags $(M=N=2)$</th>
<th>(3) Two lags (post only) $(M=1, N=2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>0.588*** (0.037)</td>
<td>0.829*** (0.057)</td>
<td>0.607*** (0.040)</td>
</tr>
<tr>
<td>Change $(\rho_{1,d})$</td>
<td>0.269*** (0.042)</td>
<td>0.184*** (0.068)</td>
<td>0.402*** (0.056)</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>--</td>
<td>-0.127** (0.054)</td>
<td>--</td>
</tr>
<tr>
<td>Change $(\rho_{2,d})$</td>
<td>--</td>
<td>-0.050 (0.066)</td>
<td>-0.172*** (0.038)</td>
</tr>
</tbody>
</table>

| Half-life: Bretton Woods | 1.34 (1.08, 1.61)  | 2.27 (1.77, 2.84) | 1.43 (1.13, 1.72) |
| (5%, 95%)               | (1.14, 1.55)       | (1.86, 2.70)      | (1.20, 1.66)      |
| Half-life: Post-Bretton Woods | 4.51 (3.52, 5.62) | 4.33 (3.47, 5.23) | 4.34 (3.50, 5.32) |
| (5%, 95%)               | (3.71, 5.35)       | (3.66, 4.96)      | (3.66, 5.04)      |
| (10%, 90%)              |                      |                      |                        |

Note: Estimates are based on the equation \( q_{j,t} = c_j + \sum_{m=1}^{M} \rho_{m}(q_{j,t-m}) + \sum_{n=1}^{N} \rho_{n,d}(d_{t}q_{j,t-n}) + \epsilon_{j,t} \) along with cross-sectional means of the dependent and all explanatory variables and the regime shift term \( d \). Half-life in years are calculated from simulated impulse responses derived from the parameter estimates. Bias correction is carried out via the Kilian (1998) bootstrap method with 1000 iterations. Standard errors in parentheses and p value are derived using the double-bootstrap method of Kilian (1998). *** indicates statistical significance at the 10% level, ** at 5%, * at 10%.
Table 4
Robustness Checks Using Data from Taylor (2002)

<table>
<thead>
<tr>
<th></th>
<th>Half-life: Bretton Woods</th>
<th>Half-life: Post-Bretton Woods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taylor data and methodology (as reported in his paper)</td>
<td>1.5</td>
<td>2.1</td>
</tr>
<tr>
<td><strong>Our Methodology:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor dataset</td>
<td>1.5</td>
<td>2.1</td>
</tr>
<tr>
<td>Excluding developing countries</td>
<td>1.9</td>
<td>3.1</td>
</tr>
<tr>
<td>Also extending data to 2011</td>
<td>1.9</td>
<td>3.6</td>
</tr>
<tr>
<td>Also using period averages</td>
<td>1.5</td>
<td>5.1</td>
</tr>
<tr>
<td>Also using our regime dates</td>
<td>2.9</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Note: Table 4 reports half-lives, in years, estimated from alternative constructions of the data set which mimic features of the data used in Taylor (2002).
<table>
<thead>
<tr>
<th>Coefficient Estimates:</th>
<th>(1) Bretton Woods</th>
<th>(2)</th>
<th>(3) Post-Bretton Woods</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of adjustment ($\rho$)</td>
<td>$-0.223^{***}$</td>
<td>$-0.102^{**}$</td>
<td>$-0.168^{***}$</td>
<td>$-0.049^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.100)</td>
<td>(0.050)</td>
<td>(0.029)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Dynamic response to $e$ ($\mu^e$)</td>
<td>$0.212^{**}$</td>
<td>$-0.143^{*}$</td>
<td>$0.183^{***}$</td>
<td>$-0.026$</td>
</tr>
<tr>
<td></td>
<td>(0.115)</td>
<td>(0.066)</td>
<td>(0.050)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>Dynamic response to $p$ ($\mu^p$)</td>
<td>$-0.322^{**}$</td>
<td>$0.303^{***}$</td>
<td>$-0.329^{**}$</td>
<td>$0.555^{***}$</td>
</tr>
<tr>
<td></td>
<td>0.151</td>
<td>0.087</td>
<td>0.123</td>
<td>0.073</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Half-life of $q$:</th>
<th>$e$ shock</th>
<th>$p$ shock</th>
<th>$e$ shock</th>
<th>$p$ shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5%, 95%)</td>
<td>2.03</td>
<td>1.56</td>
<td>3.05</td>
<td>3.84</td>
</tr>
<tr>
<td>(10%, 90%)</td>
<td>(0.85, 4.33)</td>
<td>(0.80, 2.80)</td>
<td>(2.49, 3.70)</td>
<td>(2.10, 5.92)</td>
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<tr>
<td></td>
<td>(1.02, 3.40)</td>
<td>(0.88, 2.40)</td>
<td>(2.61, 3.54)</td>
<td>(2.45, 5.38)</td>
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</tbody>
</table>

Note: The table reports estimates for the system:

$\Delta e_{jt} = \alpha_{e,j} + \rho_e (\Delta e_{j,t-1}) + \mu^e_e (\Delta e_{j,t-1}) + \zeta^e_{jt}$

$\Delta p_{jt} = \alpha_{p,j} + \rho_p (\Delta p_{j,t-1}) + \mu^p_e (\Delta e_{j,t-1}) + \mu^p_p (\Delta p_{j,t-1}) + \zeta^p_{jt}$

along with cross-sectional means of all dependent and explanatory variables. Bias correction is carried out via the Kilian (1998) bootstrap method with 1000 iterations. Standard errors in parentheses and p values are derived using the double-bootstrap method of Kilian (1998). *** indicates statistical significance at the 10% level, ** at 5%, * at 10%.
Table 6
Vector Error Correction Estimates and Half-lives in Nested Model

<table>
<thead>
<tr>
<th>Coefficient Estimates:</th>
<th>(1) e equation</th>
<th>(2) p equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of adjustment ($\rho$)</td>
<td>-0.330*** (0.063)</td>
<td>-0.079*** (0.025)</td>
</tr>
<tr>
<td>Change in speed of adjust. ($\rho_d$)</td>
<td>0.169*** (0.068)</td>
<td>0.030 (0.028)</td>
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<tr>
<td>Dynamic response to $e$ ($\mu\Delta\epsilon$)</td>
<td>0.261*** (0.068)</td>
<td>-0.086*** (0.028)</td>
</tr>
<tr>
<td>Change in dynamics to $e$ ($\mu\Delta\delta$)</td>
<td>-0.091 (0.082)</td>
<td>0.052* (0.035)</td>
</tr>
<tr>
<td>Dynamic response to $p$ ($\mu\Delta\delta$)</td>
<td>-0.269* (0.153)</td>
<td>0.285*** (0.086)</td>
</tr>
<tr>
<td>Change in dynamic to $p$ ($\mu\Delta\delta$)</td>
<td>-0.066 (0.183)</td>
<td>0.267*** (0.103)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Half-life of $q$:</th>
<th>e shock</th>
<th>p shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bretton Woods</td>
<td>1.82</td>
<td>1.34</td>
</tr>
<tr>
<td>(5%, 95%)</td>
<td>(1.41, 2.33)</td>
<td>(0.79, 2.14)</td>
</tr>
<tr>
<td>(10%, 90%)</td>
<td>(1.49, 2.20)</td>
<td>(0.85, 1.91)</td>
</tr>
<tr>
<td>Post-Bretton Woods</td>
<td>3.06</td>
<td>3.93</td>
</tr>
<tr>
<td>(5%, 95%)</td>
<td>(2.42, 3.81)</td>
<td>(2.44, 5.68)</td>
</tr>
<tr>
<td>(10%, 90%)</td>
<td>(2.54, 3.63)</td>
<td>(2.67, 5.30)</td>
</tr>
</tbody>
</table>

Note: The table reports estimates for the system:
$$\Delta e_{jt} = \alpha_{e,j} + \rho_{e,j-1} + \rho_{e,d} \left(d_q e_{j,t-1}\right) + \mu_{e} \Delta e_{j,t-1} + \mu_{e,d} \left(d_q \Delta e_{j,t-1}\right) + \mu_{p} \Delta p_{j,t-1} + \mu_{p,d} \left(d_q \Delta p_{j,t-1}\right) + \zeta_{e,j,t}$$
$$\Delta p_{jt} = \alpha_{p,j} + \rho_{p,j-1} + \rho_{p,d} \left(d_q p_{j,t-1}\right) + \mu_{p} \Delta e_{j,t-1} + \mu_{p,d} \left(d_q \Delta e_{j,t-1}\right) + \mu_{p} \Delta p_{j,t-1} + \mu_{p,d} \left(d_q \Delta p_{j,t-1}\right) + \zeta_{p,j,t}$$

along with cross-sectional means of all dependent and explanatory variables and the regime shift dummy $d_q$. Bias correction is carried out via the Kilian (1998) bootstrap method with 1000 iterations. Standard errors in parentheses and p values are derived using the double-bootstrap method of Kilian (1998). *** indicates statistical significance at the 10% level, ** at 5%, * at 10%.
<table>
<thead>
<tr>
<th>Parameter change:</th>
<th>(1) Change in var(q) (in %)</th>
<th>(2) Change in half-life of real exchange rate (in years)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange rate equation:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed of adjustment in e ($\rho_e$)</td>
<td>9.47</td>
<td>0.55</td>
<td>0.36</td>
</tr>
<tr>
<td>Dynamic response by e to e ($\mu_{ee}$)</td>
<td>-2.85</td>
<td>-0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>Dynamic response by e to p ($\mu_{ep}$)</td>
<td>-0.20</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Price equation:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed of adjustment in p ($\rho_p$)</td>
<td>9.52</td>
<td>0.48</td>
<td>0.33</td>
</tr>
<tr>
<td>Dynamic response by p to e ($\mu_{pe}$)</td>
<td>11.79</td>
<td>0.35</td>
<td>-0.07</td>
</tr>
<tr>
<td>Dynamic response by p to p ($\mu_{pp}$)</td>
<td>12.75</td>
<td>-0.21</td>
<td>1.35</td>
</tr>
</tbody>
</table>

Note: The table reports the change in variance and half-lives of the real exchange rate, relative to the Bretton Woods benchmark, for the VECM model under different combinations of parameters taken from Table 5. The column on the left indicates which parameter is set at its post-Bretton Woods value; all other parameter values are taken from the Bretton Woods estimation. Simulations are based on the system:

$$\Delta e_{jt} = \alpha_{e,j} + \rho_e(q_{e,j-1}) + \mu_{ee}(\Delta e_{j,t-1}) + \mu_{ep}(\Delta p_{j,t-1}) + \varphi_{e,j}$$

$$\Delta p_{jt} = \alpha_{p,j} + \rho_p(q_{p,j-1}) + \mu_{pe}(\Delta e_{j,t-1}) + \mu_{pp}(\Delta p_{j,t-1}) + \varphi_{p,j}.$$
Figure 1 plots impulse responses of the real exchange rate in annual data to a one-standard deviation shock to the real exchange rate, based upon CCEP estimates of the autoregression equation (2) in the panel dataset, $M = 1$ and 2. Vertical axes measure percent deviations; horizontal axis measures years after the shock.
Figure 2 plots impulse responses in annual data to a one-standard deviation shock to the nominal exchange rate, based upon CCEP estimates of the VECM model in equation (6) in the panel dataset. Vertical axes measure percent deviations. Horizontal axes measure years after the shock.
Figure 3 plots impulse responses in annual data to a one-standard deviation shock to the ratio of national price levels, based upon CCEP estimates of the VECM model in equation (6) in the panel dataset. Vertical axes measure percent deviations. Horizontal axes measure years after the shock.
The main text decomposes the variance of the real exchange rate into the variance of the innovation and a function of autoregressive terms. Equation (5) states this decomposition for the case of an autoregression with two lags. This section of the appendix explains how we derived this decomposition.

Rewrite the second-order autoregressive equation with autoregressive coefficients $\rho_1$ and $\rho_2$ as a two-equation first-order vector autoregression $y_t = Ay_{t-1} + \zeta_t$, where $y_t = [q_t, q_{t-1}]$, $\zeta_t = [\varepsilon_t, 0]'$, and $A = \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix}$. The covariance matrix of the residuals can be defined as

$$\Omega = E[\zeta_t, \zeta_t'] = \begin{bmatrix} \text{var}(\varepsilon) & 0 \\ 0 & 0 \end{bmatrix}.$$  Following Chow (1980), the contemporaneous covariance can be written as follows:

$$E[y_t y_t'] = \Omega + \sum_{j=1}^{\infty} [BD'B^{-1}] \Omega [BD'B^{-1}]',$$

where $D$ is the diagonal matrix of eigenvalues and $B$ is the matrix of eigenvectors of $A$. This can be computed:

$$\Omega + BB'$$

where the typical element $(i,j)$ of $K$ is

$$K_{ij} = \frac{(1-d_i)(1-d_j)M_{ij}}{1-d_id_j}$$

and where

$$M = B^{-1}\Omega[B^{-1}]'.$
The variance of the real exchange rate is element (1,1) of the resulting matrix. This system was solved analytically for equation (4) using Matlab’s symbolic toolbox.
The main text claims that results are robust when the sample is limited to countries that are homogeneous in the number of lags, as indicated by the AICC tests for optimal lag length. This section of the appendix reports estimates for these subsamples. The first two tables (Table A1 and Table A2) study the 14 countries for which the optimal lag length is one lag, both for the Bretton Woods period and the post-Bretton Woods period. These countries are: Australia, Austria, Canada, Finland, France, Germany, Greece, Ireland, Japan, Netherlands, Norway, Spain, Sweden, Switzerland. The second two tables (Table A3 and Table A4) study the 5 countries for which the optimal lag length is one lag for the Bretton Woods period and two for the post-Bretton Woods period. These countries are U.K., Belgium, Denmark, Portugal, and New Zealand.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>one lag</td>
<td></td>
</tr>
<tr>
<td>Bretton Woods</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post-Bretton Woods</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>0.794***</td>
<td>0.879***</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Half-life of $q$:</td>
<td>2.89</td>
<td>5.36</td>
</tr>
<tr>
<td>(5%, 95%)</td>
<td>(1.62, 4.55)</td>
<td>(3.84, 6.98)</td>
</tr>
<tr>
<td>(10%, 90%)</td>
<td>(1.80, 4.00)</td>
<td>(4.17, 6.61)</td>
</tr>
</tbody>
</table>

Note: The sample is restricted to the fourteen countries for which an AICC specification test indicates an optimal lag length of one lag. Results are comparable to those in table 2 in the main text. Estimates are based on the equation $q_{jt} = c_j + \sum_{m=1}^{M} \rho_m (q_{j,t-m}) + \varepsilon_{j,t}$ along with cross-sectional means of all dependent and explanatory variables. Half-lives in years are calculated from simulated impulse responses derived from the parameter estimates. Bias correction is carried out via the Kilian (1998) bootstrap method with 1000 iterations. Standard errors in parentheses and p value are derived using the double-bootstrap method of Kilian (1998). *** indicates statistical significance at the 1% level, ** at 5%, * at 10%.
Table A2.
AR(1) estimation for panel of 14 countries with BW and Post periods nested

<table>
<thead>
<tr>
<th></th>
<th>One lag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(M=N=1)</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.636***</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
</tr>
<tr>
<td>Change ($\rho_{1,d}$)</td>
<td>0.248***</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
</tr>
<tr>
<td>Half-life: Bretton Woods</td>
<td>1.57</td>
</tr>
<tr>
<td>(5%, 95%)</td>
<td>(1.21, 1.96)</td>
</tr>
<tr>
<td>(10%, 90%)</td>
<td>(1.29, 1.87)</td>
</tr>
<tr>
<td>Half-life: Post-Bretton Woods</td>
<td>5.56</td>
</tr>
<tr>
<td>(5%, 95%)</td>
<td>(3.91, 7.49)</td>
</tr>
<tr>
<td>(10%, 90%)</td>
<td>(4.23, 7.00)</td>
</tr>
</tbody>
</table>

Note: The sample is restricted to the fourteen countries for which an AICC specification test indicates an optimal lag length of one lag. Results are comparable to those in table 3 in the main text. Estimates are based on the equation $g_{j,t} = c_j + \sum_{m=1}^{M} \rho_m (q_{j,t-m}) + \sum_{n=1}^{N} \rho_{n,d} (d_i q_{j,t-n}) + \epsilon_{j,t}$ along with cross-sectional means of the dependent and all explanatory variables and the regime shift term $d_i$. Half-life in years are calculated from simulated impulse responses derived from the parameter estimates. Bias correction is carried out via the Kilian (1998) bootstrap method with 1000 iterations. Standard errors in parentheses and p value are derived using the double-bootstrap method of Kilian (1998). *** indicates statistical significance at the 1% level, ** at 5%, *at 10%.
Table A3.
AR estimation for panel of 5 countries with mixed lags
(analogous to Table 2 in paper)

<table>
<thead>
<tr>
<th></th>
<th>(1) one lag</th>
<th>(2) two lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bretton Woods</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.711***</td>
<td>0.926***</td>
</tr>
<tr>
<td></td>
<td>(0.084)</td>
<td>(0.061)</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>--</td>
<td>-0.282***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.058)</td>
</tr>
<tr>
<td>Half-life of $q$:</td>
<td>2.17</td>
<td>2.25</td>
</tr>
<tr>
<td>(5%, 95%)</td>
<td>(1.19, 3.58)</td>
<td>(1.85, 2.67)</td>
</tr>
<tr>
<td>(10%, 90%)</td>
<td>(1.37, 3.08)</td>
<td>(1.93, 2.58)</td>
</tr>
</tbody>
</table>

Note: The sample is restricted to the fourteen countries for which an AICC specification test indicates an optimal lag length of one lag in the Bretton Woods period and two lags in the Post-Bretton Woods period. Results are comparable to those in table 2 in the main text. Estimates are based on the equation $q_{j,t} = c_j + \sum_{m=1}^{M} \rho_m (q_{j,t-m}) + \epsilon_{j,t}$ along with cross-sectional means of all dependent and explanatory variables. Half-lives in years are calculated from simulated impulse responses derived from the parameter estimates. Bias correction is carried out via the Kilian (1998) bootstrap method with 1000 iterations. Standard errors in parentheses and p value are derived using the double-bootstrap method of Kilian (1998). *** indicates statistical significance at the 1% level, ** at 5%, * at 10%.
Table A4.
AR estimation for panel of 5 countries with BW and Post periods nested, mixed lags
(analogous to table 3 in paper)

<table>
<thead>
<tr>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two lags (post only)</td>
</tr>
<tr>
<td>( M=1, N=2 )</td>
</tr>
</tbody>
</table>

| \( \rho_1 \) | 0.747*** |
| | (0.077) |
| Change (\( \rho_{1,d} \)) | 0.189** |
| | (0.102) |
| \( \rho_2 \) | -- |
| Change (\( \rho_{2,d} \)) | -0.273*** |
| | (0.061) |

| Half-life: Bretton Woods | 2.54 |
| (5%, 95%) | (1.42, 4.27) |
| (10%, 90%) | (1.61, 3.66) |

| Half-life: Post-Bretton Woods | 2.36 |
| (5%, 95%) | (1.88, 2.88) |
| (10%, 90%) | (1.97, 2.74) |

Note: The sample is restricted to the fourteen countries for which an AICC specification test indicates an optimal lag length of one lag in the Bretton Woods period and two lags in the Post-Bretton Woods period. Results are comparable to those in table 3 in the main text. Estimates are based on the equation

\[
q_{j,t} = c_j + \sum_{m=1}^{M} \rho_m (q_{j,t-m}) + \sum_{n=1}^{N} \rho_{n,d} (d_t q_{j,t-n}) + \epsilon_{j,t}
\]

along with cross-sectional means of the dependent and all explanatory variables and the regime shift term \( d_t \). Half-life in years are calculated from simulated impulse responses derived from the parameter estimates. Bias correction is carried out via the Kilian (1998) bootstrap method with 1000 iterations. Standard errors in parentheses and p value are derived using the double-bootstrap method of Kilian (1998). *** indicates statistical significance at the 1% level, ** at 5%, * at 10%.